

A Production Inventory Model with Shortages, Fuzzy Preparation Time, Variable Production and Demand

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ABSTRACT: A production inventory model is formulated for a single item. Here, demand varies with the on-hand inventory level and production price. Shortages are allowed and fully backlogged. The time gap between the decision and actual commencement of production is termed as “preparation time” and is assumed to be crisp/imprecise in nature. The set-up cost depends on preparation time. The fuzzy preparation time is reduced to a crisp interval preparation time using nearest interval approximation and following the interval arithmetic, the reduced problem is converted to a multi-objective optimization problem. Mathematical analysis has been made for single objective crisp model (Model-I). Numerical illustration have been made for both crisp (Model-I) and fuzzy (Model-II) models. Model-I is solved by generalized reduced gradient technique and multi-objective model (Model-II) by Global Criteria Method. Sensitivity analysis have been made for some parameters of Model-I.

KEYWORDS: Inventory ,Deterioration ,shortages,Demand, fuzzy variable,Preparation Time, Global Criteria Method

I.INTRODUCTION

The production inventory model for a single item, is studied. In classical Inventory Models demand is normally assumed to be constant or known advance practically, due to market competition demand is not known as a fixed one with respect to a seller. More over demand of an item depends indirectly with the unit production cost because the production cost fixes selling price. Existence of Lead-time is a natural phenomenon So far, most of the researchers have dealt with either constant or stochastic lead-time. For the first time, Mahapatra and Maiti [6,7] formulated and solved production inventory models for a deteriorating/breakable item with imprecise preparation time. Shortages are allowed and backlogged fully. Preparation time in production of the new consignment is allowed and crisp/fuzzy in nature. The setup cost is dependent on preparation time. The crisp problem for minimizing average inventory cost is solved by generalized reduced gradient method. The problem minimizing average total cost with fuzzy preparation time is converted to a multi-objective minimization problem with the help of interval arithmetic. Then it is solved by global criteria method to get the optimal solution. Mathematical derivation and analysis also have been made for both single and multiobjective models. Further, the sensitivity analysis is included and two numerical illustration are provided.

II.FORMULATION OF THE MULTI-OBJECTIVE PROBLEM

A general non-linear objective function with some interval valued parameters is of the following form:

$$\text{Min } Z(x) = \sum_{i=1}^k C_i \prod_{j=1}^n x_j^{a_{ij}} \quad (1)$$

Subject To Constraints

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(A High Impact Factor, Monthly, Peer Reviewed Journal)

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Vol. 7, Issue 4, April 2018

$$\sum_{i=1}^k A_i x_j \leq B_j, \quad x_j > 0, \quad (j = 1, 2, \dots, n), \quad x = (x_1, x_2, \dots, x_n) \quad (2)$$

where $C_i = [c_{L_i}, c_{R_i}]$, $A_i = [a_{L_i}, a_{R_i}]$, $B_j = [b_{L_j}, b_{R_j}]$

If the right and left limits of the intervals $Z(x)$ are $Z_R(x)$, $Z_L(x)$ and center $Z_C(x)$ respectively, then the objective function of the problem (1) may be elicited as,

$$Z_R(x) = \sum_{i=1}^k C_{L_i} \prod_{j=1}^n x_j^{a_{ij}}, \quad Z_L(x) = \sum_{i=1}^k C_{R_i} \prod_{j=1}^n x_j^{a_{ij}}, \quad Z_C(x) = \frac{1}{2} [Z_R(x) + Z_L(x)]$$

Thus the problem (2) is transformed into

$$\text{Min} \{Z_R, Z_C\}$$

subject to constraints

$$\sum_{i=1}^k a_{L_i} x_j \leq b_{R_j}$$

$$\sum_{i=1}^k a_{R_i} x_j \leq b_{L_j}, \quad x_j > 0, \quad (j = 1, 2, \dots, n), \quad \bar{x} = (x_1, x_2, \dots, x_n)$$

The Nearest Interval Approximation

A fuzzy number center approximated by crisp quantity. Suppose \tilde{A} and \tilde{B} are two fuzzy number with α -cuts are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$.

Then the distance between \tilde{A} and \tilde{B} is given by

$$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha} \quad (3)$$

Let \tilde{A} be a fuzzy number a closed interval $c_d(\tilde{A})$ which is nearest to \tilde{A} with respect to metric d is given by

$(c_d(\tilde{A}))_\alpha = [c_L(\alpha), c_R(\alpha)]$. Then using the equation (3)

$$d(\tilde{A}, c_d(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - c_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - c_R(\alpha))^2 d\alpha} \quad (4)$$

with respect to c_L and c_R

In order to minimize $d(\tilde{A}, c_d(\tilde{A}))$ it is sufficient to minimize the function

$$D(c_L, c_R) = (d^2(\tilde{A}, c_d(\tilde{A})))$$

$$\frac{\partial D(c_L, c_R)}{\partial c_R} = -2 \int_0^1 A_R(\alpha) d\alpha + 2c_R \quad (5)$$

$$\frac{\partial D(c_L, c_R)}{\partial c_L} = -2 \int_0^1 A_L(\alpha) d\alpha + 2c_L \quad (6)$$

$$\text{Solving } \frac{\partial D(c_L, c_R)}{\partial c_L} = 0, \quad \frac{\partial D(c_L, c_R)}{\partial c_R} = 0$$

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 7, Issue 4, April 2018

Using the equation (5) and (6) yields ,

$$C_L^* = \int_0^1 A_L(\alpha) d\alpha \text{ and } C_R^* = \int_0^1 A_R(\alpha) d\alpha$$

$$\text{Since } \frac{\partial D^2(C_L^*, C_R^*)}{\partial C_L^2} = 2 > 0, \quad \frac{\partial D^2(C_L^*, C_R^*)}{\partial C_R^2} = 2 > 0$$

$$H(C_L^*, C_R^*) = \frac{\partial D^2(C_L^*, C_R^*)}{\partial C_L^2} \cdot \frac{\partial D^2(C_L^*, C_R^*)}{\partial C_R^2} - \left(\frac{\partial D^2(C_L^*, C_R^*)}{\partial C_L \cdot \partial C_R} \right)^2 = 4 > 0$$

so $D(C_L, C_R)$ i.e. $d(\tilde{A}, c_d(\tilde{A}))$ is global minimum. Therefore, the interval

$$c_d(\tilde{A}) = \left[\int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_R(\alpha) d\alpha \right]$$

is the nearest interval approximation of fuzzy number \tilde{A} with respect to metric d .

Let $\tilde{A} = (a_1, a_2, a_3)$ be fuzzy number. The α level interval of \tilde{A} is defined as $(\tilde{A})_\alpha = [A_L(\alpha), A_R(\alpha)]$.

When \tilde{A} is a Triangular fuzzy number. Then $A_L(\alpha) = a_1 + \alpha(a_2 - a_1)$ and $A_R(\alpha) = a_3 + \alpha(a_3 - a_2)$

By nearest interval approximation method lower and upper limits of the interval are respectively calculated as follows :

$$\begin{aligned} C_L &= \int_0^1 A_L(\alpha) d\alpha \\ &= \int_0^1 [a_1 + \alpha(a_2 - a_1)] (\alpha) d\alpha \\ &= a_1 \int_0^1 d\alpha + (a_2 - a_1) \int_0^1 \alpha d\alpha = \frac{1}{2}(a_2 + a_1) \end{aligned}$$

$$\begin{aligned} C_R &= \int_0^1 A_R(\alpha) d\alpha \\ &= \int_0^1 [a_3 + \alpha(a_3 - a_2)] (\alpha) d\alpha \\ &= a_3 \int_0^1 d\alpha + (a_3 - a_2) \int_0^1 \alpha d\alpha \\ &= \frac{1}{2}(a_2 + a_3) \end{aligned}$$

Therefore, interval number considering \tilde{A} as a Traingular fuzzy number is $\left[\frac{(a_1 + a_2)}{2}, \frac{(a_2 + a_3)}{2} \right]$.

III. ASSUMPTIONS AND NOTATIONS FOR THE PROPOSED MODEL

The following notations and assumptions are used in developing the model.

Assumptions :

- Production system involves only one non-deteriorating item.
- Shortage are allowed and fully backlogged
- Time horizon is infinite.

Notations :

- $q(t)$: Inventory level at any time t .
- $Q_m(Q_s)$: The maximum inventory (shortages) level in a cycle
- \bar{L} : Preparation time for the next production which is a fuzzy number
- C_3 : Setup cost which is of the form $C_3 = C_{30} - C_{31}L^{-\gamma}$
- p : Production cost per unit item.
- D : Rate of Demand which depends on production price

$$\text{and stock i.e., } D(p, q) = \begin{cases} p^{-\epsilon} (a + b q^{1-\beta}) & \text{if } q > 0 \\ a p^{-\epsilon} & \text{if } q \leq 0 \end{cases}$$

where a, b are positive real constants.

- t_0 : cycle length of a cycle.
- C_1 : Holding cost per unit per unit time.
- C_2 : Shortage cost per unit per unit time.
- t' : Reproduction time i.e., time when production for next cycle is decided.

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Vol. 7, Issue 4, April 2018

- K : Production rate which is of the form $K=\mu D$ and $\mu \geq 1$

IV. MATHEMATICAL FORMULATION

Production Inventory System involves only one time a cycle start with shortages at time $t = 0$ and at time t_1 maximum shortages level Q_s is reached and at that time production process starts to backlog the shortage quantities fully and after time t_2 the shortages reached to zero. Inventory accumulates up to time t_3 of amount Q_m . At that time production process stopped, accumulated inventory declines due to demand and reaches to zero at time t_0 .

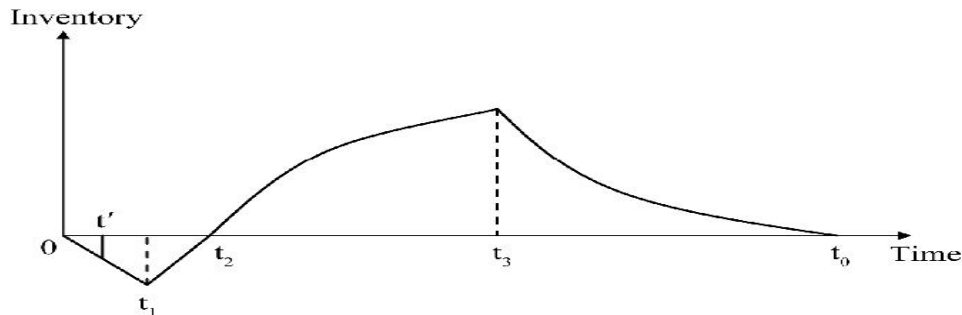


Figure (4.1) Inventory begins with backlogged shortages and end with no inventory .

The differential equation governing the stock status of this model are derived as follows :

$$\frac{dq}{dt} = \begin{cases} -D & \text{if } 0 \leq t \leq t_2 \\ K - D & \text{if } t_1 \leq t \leq t_2 \\ K - D & \text{if } t_2 \leq t \leq t_3 \\ -D & \text{if } t_3 \leq t \leq t_0 \end{cases} \quad (7)$$

with the boundary condition $q(t_1) = q(t_2) = q(t_0) = 0$ These equations can be written in the following forms .

$$\frac{dq}{dt} = \begin{cases} -ap^{-\varepsilon} & \text{if } 0 \leq t \leq t_1 \\ (\mu - 1)ap^{-\varepsilon} & \text{if } t_1 \leq t \leq t_2 \\ (\mu - 1)p^{-\varepsilon} (a + bq^{1-\beta}) & \text{if } t_2 \leq t \leq t_3 \\ -p^{-\varepsilon} (a + bq^{1-\beta}) & \text{if } t_3 \leq t \leq t_0 \end{cases} \quad (8)$$

for simplicity take $\beta = 0$, solving first and second equations of differential equation (8) in the intervals $0 \leq t \leq t_1$ and $t_1 \leq t \leq t_2$ respectively with the help of the boundary conditions $q(t_1) = 0 = q(t_2)$ we get

$$q(t) = \begin{cases} -ap^{-\varepsilon}t & \text{from the first equation of (8)} \\ -(\mu - 1)ap^{-\varepsilon}(t_2 - t) & \text{from the second equation of (8)} \end{cases}$$

$$Q_s = \begin{cases} -ap^{-\varepsilon}t & \text{from the first equation of (8)} \\ -(\mu - 1)ap^{-\varepsilon}(t_2 - t) & \text{from the second equation of (8)} \end{cases}$$

so at time $t = t_1$

$$\begin{aligned} -ap^{-\varepsilon}t_1 &= -(\mu - 1)ap^{-\varepsilon}t_2 + (\mu - 1)ap^{-\varepsilon}t_1 \\ -ap^{-\varepsilon}t_1 - (\mu - 1)ap^{-\varepsilon}t_1 &= -(\mu - 1)ap^{-\varepsilon}t_2 \\ ap^{-\varepsilon}t_1(-1 - \mu + 1) &= -(\mu - 1)ap^{-\varepsilon}t_2 \\ \mu t_1 &= (\mu - 1)t_2 \end{aligned} \quad (9)$$

Similarly solving third and fourth differential equations of in the interval $t_2 \leq t \leq t_3$ and $t_3 \leq t \leq t_0$ respectively with the help of the boundary conditions $q(t_2) = 0 = q(t_0)$. Then

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Vol. 7, Issue 4, April 2018

$$q(t) = \begin{cases} \frac{a}{b} \{e^{x(t-t_2)} - 1\} & \text{from the third equation of (8)} \\ \frac{a}{b} \{e^{bp^{-\varepsilon}(t_0-t)} - 1\} & \text{from the fourth equation of (8)} \end{cases}$$

$$Q_m = \begin{cases} \frac{a}{b} \{e^{x(t_3-t_2)} - 1\} & \text{from the third equation of (8)} \\ \frac{a}{b} \{e^{bp^{-\varepsilon}(t_0-t_3)} - 1\} & \text{from the fourth equation of (8)} \end{cases}$$

where $x = (\mu - 1)bp^{-\varepsilon}$ at time $t = t_3$

$$\frac{a}{b} \{e^{x(t_3-t_2)} - 1\} = \frac{a}{b} \{e^{bp^{-\varepsilon}(t_0-t_3)} - 1\}$$

$t_3 = \frac{t_0}{\mu} + t_1$ using the relation (8)

Total Shortage cost (S_C) is given by

$$S_C = -c_2 \left[\int_0^{t_1} q(t)dt + \int_{t_1}^{t_2} q(t)dt \right]$$

$$= \frac{1}{2}c_2 ap^{-\varepsilon}t_1^2 + \frac{1}{2}c_2 (\mu - 1)ap^{-\varepsilon}(t_2 - t_1)^2$$

Where $t_1 = L + t'$, $t_2 = \frac{\mu}{\mu-1}t_1$, $t_3 = \frac{t_0}{\mu} + t_1$

Total Holding cost (H_c) during the interval $[t_2, t_3]$ and $[t_3, t_0]$ is derived as follows

$$H_c = c_1 \left[\int_{t_2}^{t_3} q(t)dt + \int_{t_3}^{t_0} q(t)dt \right]$$

$$= \frac{c_1 a}{b} \left[\frac{1}{x} \{e^{x(t_3-t_2)} - 1\} + t_2 - \frac{c_1 a}{b} \left[\frac{1}{bp^{-\varepsilon}} \{1 - e^{bp^{-\varepsilon}(t_0-t_3)}\} + t_0 \right] \right]$$

Total Production cost (P_c)

$$P_c = p \int_{t_1}^{t_2} \mu ap^{-\varepsilon} dt + p \int_{t_2}^{t_3} \mu p^{-\varepsilon} (a + bq) dt$$

$$= p^{1-\varepsilon} \mu a (t_2 - t_1) + \frac{1}{x} p^{1-\varepsilon} \mu a \{e^{x(t_3-t_2)} - 1\}$$

Average total cost (ATC)

$$ATC = \frac{1}{t_0} (\text{Holding cost} + \text{setup cost} + \text{Shortage cost} + \text{Production Cost}) = \frac{TC}{t_0}$$

$$TC = \left[\frac{c_1 a}{bx} \{e^{x(t_3-t_2)} - 1\} + \frac{c_1 a}{b} t_2 - \frac{c_1 a}{b^2} \frac{1}{p^{-\varepsilon}} \{1 - e^{bp^{-\varepsilon}(t_0-t_3)}\} - \frac{t_0 c_1 a}{b} + C_{30} - C_{31} L^\gamma + \frac{1}{2} c_2 ap^{-\varepsilon} t_1^2 + \frac{1}{2} c_2 (\mu - 1) ap^{-\varepsilon} (t_2 - t_1) + p^{1-\varepsilon} \mu a (t_2 - t_1) + \frac{1}{x} p^{1-\varepsilon} \mu a \{e^{x(t_3-t_2)} - 1\} \right]$$

Crisp Model (Model-I)

The proposed model can be stated as

$$\text{Minimize ATC}(t', t_0) = \frac{1}{t_0} \left[\frac{c_1 a}{bx} \{e^{x(t_3-t_2)} - 1\} + \frac{c_1 a}{b} t_2 - \frac{c_1 a}{b^2} \frac{1}{p^{-\varepsilon}} \{1 - e^{bp^{-\varepsilon}(t_0-t_3)}\} - \frac{t_0 c_1 a}{b} + C_{30} - C_{31} L^\gamma + \frac{1}{2} c_2 ap^{-\varepsilon} t_1^2 + \frac{1}{2} c_2 (\mu - 1) ap^{1-\varepsilon} \mu a (t_2 - t_1) + \frac{1}{x} p^{1-\varepsilon} \mu a \{e^{x(t_3-t_2)} - 1\} \right] \quad (10)$$

Fuzzy model (Model II)

The preparation time is a fuzzy number \tilde{L} which is replaced by an appropriate interval number $L = [L_L, L_R]$ and so. In this case t_1 is replaced by

$[L_L, L_R] + t'$. Then the crisp problem becomes a fuzzy optimization problem. Using interval arithmetic calculations the problem becomes a multi objective non linear programming problem as follows.

Minimize ATC_L, ATC_R

$$\text{Where } ATC_L = \frac{1}{t_0} [H_{CL} + C_{3L} + S_{CL} + P_{CL}] \quad (11)$$

$$ATC_R = \frac{1}{t_0} [H_{CR} + C_{3R} + S_{CR} + P_{CR}] \text{ and} \quad (12)$$

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Vol. 7, Issue 4, April 2018

using (11) and (12) we get

$$ATC_c = \frac{1}{2}(ATC_L + ATC_R) \tag{13}$$

V. MATHEMATICAL ANALYSIS

Model –I (Crisp Model)

The Objective function ATC of Model I is a function of t' and t_0 as t_1, t_2, t_3 are dependent variable.

To achieve optimal t' and t_0 . The partial derivatives of the average total cost ATC with respect to t' and t_0 are set to zero the resulting equation can be solved simultaneously to obtain the optimal t' and t_0 .

$$\frac{\partial ATC}{\partial t'} = \frac{1}{t_0} \frac{\partial TC}{\partial t'}$$

$$= \frac{1}{t_0} \left[\frac{-C_1 a}{b(\mu-1)} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} + \frac{C_1 a \mu}{b(\mu-1)} - \frac{C_1 a}{b} e^{bp^{-\epsilon} \left\{ \frac{\mu-1}{\mu} t_0 - (L+t') \right\}} + \frac{p^{1-\epsilon} \mu a}{\mu-1} - \frac{p^{1-\epsilon} \mu a}{\mu-1} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} + c_2 a p^{-\epsilon} (L+t') \right] + c_2 a p^{-\epsilon} \left(\frac{1}{\mu-1} t' + \frac{1}{\mu-1} L \right)$$

= 0

$$\frac{\partial ATC}{\partial t_0} = \frac{1}{t_0} \frac{\partial TC}{\partial t_0} - \frac{1}{t_0^2} TC$$

$$= \frac{1}{t_0} \left[\frac{-C_1 a}{b\mu} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} + \frac{C_1 a(\mu-1)}{b\mu} e^{bp^{-\epsilon} \left\{ \frac{\mu-1}{\mu} t_0 - (L+t') \right\}} - \frac{C_1 a}{b} + \frac{p^{1-\epsilon} \mu a}{\mu} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} \right] - \frac{1}{t_0^2} TC = 0 \tag{14}$$

i.e. $\frac{\partial TC}{\partial t_0} = \frac{1}{t_0} TC$ ATC can be proved as strictly convex since all the principal minors of its

Hessian matrix are strictly positive. $\frac{\partial^2 ATC}{\partial t'^2} = \frac{1}{t_0} \frac{\partial^2 TC}{\partial t'^2}$

$$\text{where } \frac{\partial^2 TC}{\partial t'^2} = \left[\frac{C_1 a x}{b(\mu-1)^2} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} + C_1 a p^{-\epsilon} e^{bp^{-\epsilon} \left\{ \frac{\mu-1}{\mu} t_0 - (L+t') \right\}} + \frac{p^{1-\epsilon} \mu a x}{(\mu-1)^2} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} + c_2 a p^{-\epsilon} + \frac{C_2 a p^{-\epsilon}}{\mu-1} \right] \tag{15}$$

$$\frac{\partial^2 ATC}{\partial t_0^2} = \frac{1}{t_0} \frac{\partial^2 TC}{\partial t_0^2} - \frac{2}{t_0^2} \frac{\partial TC}{\partial t_0} + \frac{2}{t_0^3} TC = \frac{1}{t_0} \frac{\partial^2 TC}{\partial t_0^2}$$

Using (15), where

$$\frac{\partial^2 TC}{\partial t_0^2} = \left[\frac{C_1 a x}{b\mu^2} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} + \frac{C_1 a(\mu-1)^2}{b\mu^2} p^{-\epsilon} e^{bp^{-\epsilon} \left\{ \frac{\mu-1}{\mu} t_0 - (L+t') \right\}} + \frac{p^{1-\epsilon} \mu a x}{\mu^2} e^{x \left[\frac{t_0}{\mu} - \frac{L}{\mu-1} - \frac{t'}{\mu-1} \right]} \right] \tag{16}$$

and

$$\frac{\partial^2 ATC}{\partial t'^2} * \frac{\partial^2 ATC}{\partial t_0^2} - \left\{ \frac{\partial^2 ATC}{\partial t_0 \partial t'} \right\}^2 = \frac{1}{t_0^2} \left\{ \frac{\partial^2 TC}{\partial t'^2} * \frac{\partial^2 TC}{\partial t_0^2} - \left\{ \frac{\partial^2 TC}{\partial t_0 \partial t'} \right\}^2 \right\}$$

Using (15) and (16)

$$\left\{ c_2 a p^{-\epsilon} + \frac{c_2 a p^{-\epsilon}}{\mu-1} \right\} * \frac{\partial^2 ATC}{\partial t_0^2} > 0$$

Hence, the solution of Model-I is global minimum of ATC.

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Vol. 7, Issue 4, April 2018

Model II (Fuzzy Model)

The objective Function ATC_L and ATC_R are functions of t' and t_0 as t_1, t_2, t_3 are dependent variable as of Model -I it can be seen that ATC_L and ATC_R are strictly convex.

The above problem is a strictly convex multi-objective minimization problem and hence possesses global Pareto-optimal solutions. Hence, the problem is solved by Global Criteria Method.

Global Criteria Method.

The Multi-Objective Non-linear Programming (MONLP) problems are solved by Global Criteria Method converting it to a single objective optimization problem. The solution procedure is as follows.

Step -1 :Solve the multi-objective programming problem as a single .objective problem using only one objective at a time ignoring.

Step -2 :From the above result, formulate a pay-offmatrix as follows

$$\begin{pmatrix} ATC_c^{\min} & ATC_R^{\max} \\ ATC_c^{\max} & ATC_R^{\min} \end{pmatrix}$$

Step -3. From the pay off matrix of step 2 determine the ideal objective vector say $(ATC_c^{\min}, ATC_R^{\min})$ and the corresponding value of $(ATC_c^{\max}, ATC_R^{\max})$

Here the ideal objective vector is used as a reference point .The problem is then to solve the following auxillary problem .

$$GC = \text{minimize} \left\{ \left(\frac{ATC_c(t', t_0) - ATC_c^{\min}}{ATC_c^{\max} - ATC_c^{\min}} \right)^p + \left(\frac{ATC_R(t', t_0) - ATC_R^{\min}}{ATC_R^{\max} - ATC_R^{\min}} \right)^p \right\}^{\frac{1}{p}} \quad (17)$$

Where $1 \leq p \leq \infty$ and GC is known as Global Criteria.

VI. NUMERICAL ILLUSTRATIONS

Crisp Model (Model)

The equation (10) is nonlinear in t' and t_0 and difficult to obtain the optimal values of t' and t_0 by calculus method . Hence we minimize(10) gradient based nonlinear optimization method using the optimal values of t_1, t_2, t_3, Q_s, Q_m and minimum average total cost are be obtained.

To illustrate the proposed inventory model following input data are considered.

$L = 0.6, \mu = 1.8, a = 300, b = 2, \epsilon = 0.7, \gamma = 0.5, C_1 = 1.5, p = 25, C_{30} = 2000, C_{31} = 300, C_2 = 15, t' = 0.6001609, t_0 = 6.939239$ is appropriate value. Solving the Model -I following optimal values are obtained

$$ATC = \frac{1}{t_0} (\text{Holding Cost} + \text{Setup Cost} + \text{Shortage Cost} + \text{Production Cost}) = \frac{TC}{t_0}$$

To find t_1, t_2, t_3

$$t_1 = L + t' = 0.6 + 0.6001609 = 1.2001609$$

$$t_2 = \frac{\mu}{\mu-1} t_1 = 1.3501810$$

$$t_3 = \frac{t_0}{\mu} + t_1 = 5.0552937$$

$$TC = \frac{1}{t_0} \left[\frac{C_1 a}{bx} \{ e^{x(t_3-t_2)} - 1 \} + \frac{c_1 a}{b} t_2 - \frac{c_1 a}{b^2} \frac{1}{p-\epsilon} \{ 1 - e^{bp-\epsilon(t_0-t_3)} \} \right. \\ \left. - \frac{t_0 C_1 a}{b} + C_{30} - C_{31} L^\gamma + \frac{1}{2} C_2 ap^{-\epsilon} t_1^2 + \frac{1}{2} C_2 (\mu - 1) ap^{1-\epsilon} \mu a (t_2 - t_1) + \frac{1}{x} p^{1-\epsilon} \mu a \{ e^{x(t_3-t_2)} - 1 \} \right]$$

$$TC = 9412.036817$$

$$ATC = 1356.35$$

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Vol. 7, Issue 4, April 2018

To find Q_s, Q_m

$$Q_s = \begin{cases} -ap^{-\epsilon t} & \text{if } 0 \leq t \leq t_1 \\ -(\mu - 1)ap^{-\epsilon}(t_2 - t) & \text{if } t_1 \leq t \leq t_2 \end{cases}$$

$$= \begin{cases} 37.827 & \text{if } 0 \leq t \leq 1.2 \\ 38.827 & \text{if } 1.2 \leq t \leq 1.35 \end{cases}$$

$$Q_m = \begin{cases} \frac{a}{b} \{e^{x(t_3-t_2)} - 1\} & \text{if } t_2 \leq t \leq t_3 \\ \frac{a}{b} \{e^{bp^{-\epsilon}(t_0-t_3)} - 1\} & \text{if } t_3 \leq t \leq t_0 \end{cases}$$

$$= \begin{cases} 129.626 & \text{if } 1.2 \leq t \leq 1.35 \\ 72.84892 & \text{if } 1.35 \leq t \leq 6.9 \end{cases}$$

= 72.84892

The optimal values are obtained by $ATC = 1356.35, Q_s = 37.82707, Q_m = 72.84892$.

Fuzzy Model (Model II)

To illustrate the proposed inventory Model II following input data are considered.

$L_L = 0.5, L_R = 0.8, \mu = 1.8, a = 300, b = 2, \epsilon = 0.7, \gamma = 0.5, C_1 = 1.5,$

$P = 25, C_{30} = 2000, C_{31} = 300, C_2 = 15$ in appropriate units.

For the above input data formulate a pay off matrix as follows

$$\begin{matrix} & ATC_c & ATC_R \\ \begin{pmatrix} 1364.005 & 1753.147 \\ 1385.50 & 1722.588 \end{pmatrix} \end{matrix}$$

Solving the model -II using global criteria method following optimal values are obtained

$$ATC_c = \frac{1}{2}(ATC_L + ATC_R)$$

Where $ATC_L = \frac{1}{t_0} [H_{CL} + C_{3L} + S_{CL} + P_{CL}]$

$$H_{CL} = \left[\frac{c_1 a}{b} \left[\frac{1}{x} \{e^{x(t_3-t_2)} - 1\} + t_2 - \frac{c_1 a}{b} \left[\frac{1}{bp^{-\epsilon}} \{1 - e^{bp^{-\epsilon}(t_0-t_3)}\} + t_0 \right] \right] \right]$$

$$= [225 [5.948921 \{0.8641776 + 1.3501810\} - 225 [4.7591399 - 3160.108]]]$$

$$= 3946.54$$

$$C_{3L} = C_{30} - C_{31} L^{-\gamma}$$

$$= 2000 - 300(0.5^{-0.5})$$

$$= 3072.39$$

$$S_{CL} = \frac{1}{2} c_2 ap^{-\epsilon} t_1^2 + \frac{1}{2} c_2 (\mu - 1) ap^{-\epsilon} (t_2 - t_1)^2$$

$$= 340.4889 + 28.37044$$

$$= 368.8593$$

$$P_{CL} = p^{1-\epsilon} \mu a (t_2 - t_1) + \frac{1}{x} p^{1-\epsilon} \mu a \{e^{x(t_3-t_2)} - 1\}$$

$$= 25^{0.3} (1.8)(300)(0.150021) + \frac{1}{0.1680977} (25^{0.3} (1.8)(300))$$

$$\{e^{0.1680977 (5.05529-1.3501)} - 1\}$$

$$= 212.7785 + 7291.495$$

$$= 7504.2735$$

$$ATC_L = \frac{1}{t_0} [H_{CL} + C_{3L} + S_{CL} + P_{CL}] = 1008.61$$

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Vol. 7, Issue 4, April 2018

Similarly we can find for $ATC_R = 1730.23, GC = 0.3543743$

$$ATC_c = \frac{1}{2}(ATC_L + ATC_R)$$

$$= \frac{1}{2}(1008.61 + 1730.23)$$

$$ATC_c = 1369.40$$

Hence the solution of Model –II using global criteria method following optimal values are obtained
 $ATC_L = 1008.61$, $ATC_R = 1730.23$, $ATC_c = 1369.40$, $GC = 0.3543743$

VII .SENSITIVITY ANALYSIS

Sensitivity analysis of the decision variable t' and t_0 for model –I have been made when each of the parameters a, μ, p, L being changed from -50% to +50% the relative change of ATC_c^*, Q_s^*, Q_m^* are also taken in to account and presented below.

Table7.1 Sensitivity analysis of parameter a, μ, L, p

Parameter	%change	% of ATC*	t'^*	t_0^*	Q_s	Q_m
A	-50	-42.09005	1.051922	9.5283	26.03292	54.05133
	-20	-16.43247	0.72755	7.69309	33.47436	66.22929
	+20	+16.07881	0.506768	6.37338	41.86019	78.68090
	+50	+39.71182	0.404090	5.73726	47.47090	86.34708
M	-50	No Solution	-	-	-	-
	-20	-4.761578	0.4635499	8.362338	33.52132	55.21444
	+20	+2.550066	0.6733307	6.306766	40.13326	83.86810
	+50	+4.740787	0.7361900	5.817491	42.11448	94.57485
L	-50	+0.7166992	0.9207250	7.061990	38.47522	74.44632
	-20	+0.2596661	0.7276148	6.983796	38.06201	73.42793
	+20	-0.2364422	0.4733806	6.898645	37.61337	72.32223
	+50	-0.5588499	0.2841310	6.843172	37.32184	71.60373
P	-50	-2.621729	0.2851214	5.590982	45.31972	108.9680
	-20	-1.285355	0.4873798	40.06651	40.06651	83.00596
	+20	+1.360556	0.7014259	36.10404	36.10404	65.42835
	+50	+3.408616	0.8376034	34.11446	34.11446	57.30315

From above table it is seen that ATC changes rapidly with the rapid change of the parameter a ,but these change are too slow in case of parameter μ and p . Again ATC increases in preparation time L respectively but this change is slow Which agrees with the real phenomenon.

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Vol. 7, Issue 4, April 2018

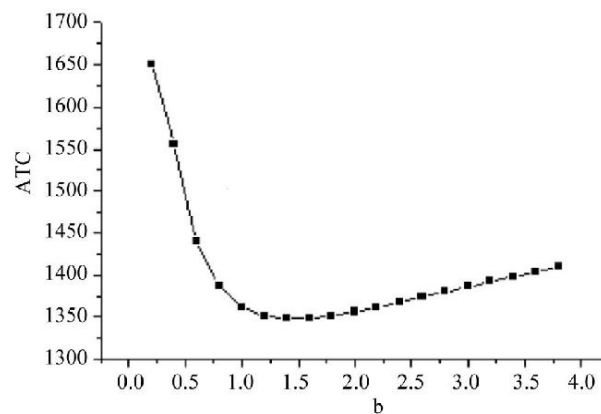


Figure (7.1) Graph of ATC versus b

Figure (7.1) shows the ATC versus b graph. From this graph it is revealed that initially ATC decreases rapidly with the increases in b and after some time it begins to increase slowly. Here it is interesting to note the effect of shape parameter b of the demand function on ATC.

VIII. CONCLUSIONS

The paper proposes a solution procedure for production inventory model with production cost and on hand inventory dependent demand rate and preparation time. Here, shortages are allowed and backlogged fully. Like lead time, time when the decision is taken for the preparation of next production run i.e. preparation time has been considered for a production inventory model. In real life, setup cost decreases with the increase of preparation time. This consideration is taken into account in Model-I. In Model-II, preparation time is taken as imprecise via a fuzzy number. The fuzzy number is described by linear/no Fuzzy number is then zzy-stochastic environment cost and inventory costs through. [4] E. A. Silver and R. Peterson, "Decision System for Inventory-linear type membership function. member describing preparation time approximated to an interval number. Following this, the problem is converted into multi-objective inventory problem where the objective functions are represented by centre and right limit of interval function which are to be minimized. To obtain the solution of the multi-objective inventory problem, Global Criteria Method has been used. The proposed demand has a broad area of applicability. Demand of a commodity decreases with the increase in production cost but increases with the increase of stock of the displayed commodity and vice versa. Here, though the formulation of the model and the solution procedure are quite general, the model is a simple production model with demand dependent production rate. The unit production cost which is assumed here to be constant, in reality, varies with the preparation time and produced quantity. Moreover, time dependent production rate, partially lost sales, inflation, etc., can be incorporated to the model to make it more realistic. Here, demand is stockdependent. The present analysis can be repeated for the dynamic demand also. Though the problem has been presented in crisp and fuzzy environment, it can also be formulated and solved in representing production probability distribution.

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